Answer all the following questions

Q1. (15 marks) A Source with 7 symbols (A, B, C, D, E, F, G) with probabilities:

1/4 , 3/16 , 3/16 , 1/8 , 1/16 ,1/16 , 1/8 respectively.

(i) What is its entropy?
(ii) Design a code using Shanon-Fano algorithm to encode the source using a binary encoded alphabet {0,1}.
(iii) Calculate the minimum length L_{min}.
(iv) Find average length of the code L_{ave} .
(v) What is the code efficiency μ?

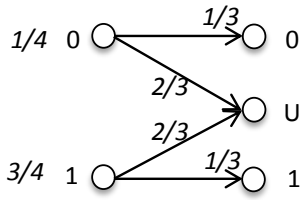
Q2. a- (10 marks) Encode the following messages using the mentioned method:

(i) abraabracadabra (LZ77)
(ii) 10000000000011000000011111 (Run length)

b- (5 marks) Decode (1,1) , (8,7) , (19,3) if it was encoded using Zero suppression.

Q.3 a- (7 marks) A Binary Erasure Channel has the following form with the mentioned input & transition probabilities. Let the input to be X and the output Y.

(i) Extract the transition Matrix.
(ii) Compute H(X).
(iii) Find: p(Y=1|X=0) , p(Y=0,X=1) , p(Y=U)



b- (8 marks) A binary symmetric channel with a transition probability of $\frac{2}{5}$ is cascaded with a second channel also with a transition probability of $\frac{2}{5}$ so that the output of the first channel feeds the second channel. Denote the input to the first channel to be X, the output of the first channel and the input to the second to be Y, and the output of the second to be Z. The source at X has a probability of a 0 as $\frac{1}{10}$.

(i) Sketch the two channels in cascade.
(ii) At the output of the first channel, find: p(Y=1|X=0) , p(Y=1,X=0) , p(Y=1)
(iii) At the output of the second channel, find: p(Z=0|X=0) , p(Z=1,X=1) , p(Z=1)

Q4.a- (6 marks) What is the minimum distance d_{min} between the codewords **00011010** , **01101001** and how many errors can be detected d_d , corrected d_c **?**

b- (9 marks) The codeword **101101000101001** was received in a system using a Hamming code with the given generator matrix :

(i) Extract the parity-check matrix.
(ii) Calculate the error vector.
(iii) Find the codeword.

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

كلية التقنية الالكترونية
College of Electronic Technology - Tripoli
طرابلس

الاجابة النموذجية للأمتحان النهائي لمادة:

نظرية المعلومات والترميز

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Q1.

$(i)\quad H(X) = -\sum_k p_k \log_2(p_k)$

$$= -\left[\frac{1}{4}\log_2\left(\frac{1}{4}\right) + \frac{3}{16}\log_2\left(\frac{3}{16}\right) + \frac{3}{16}\log_2\left(\frac{3}{16}\right) + \frac{1}{8}\log_2\left(\frac{1}{8}\right) + \frac{1}{16}\log_2\left(\frac{1}{16}\right) + \frac{1}{16}\log_2\left(\frac{1}{16}\right) + \frac{1}{8}\log_2\left(\frac{1}{8}\right)\right]$$

$$\boldsymbol{H(X) = 2.655\ bits}$$

(ii)

A	1/4	0	0			
B	3/16	0	1			
C	3/16	1	0	0		
D	1/8	1	0	1		
G	1/8	1	1	0		
E	1/16	1	1	1	0	
F	1/16	1	1	1	1	

(iii)

$$\boldsymbol{L_{min} = \frac{H(X)}{\log_2(s)} = \frac{2.655}{1} = 2.655}$$

(iv)

$$L_{ave} = \sum_i p_i\, l_i = (1/4)2 + (3/16)2 + (3/16)3 + (1/8)3 + (1/8)3 + (1/16)4 + (1/16)4$$

$$L_{ave} = 2.6875$$

(v)

$$\boldsymbol{\mu = \frac{L_{min}}{L_{ave}} = \frac{2.655}{2.6875} = 98.81\%}$$

Q2 a.

(i)

(0,0) a
(0,0)b
(0,0)r
(3,1)a
(4,3)c
(2,1)d
(7,4)

(ii)

01902265

Q2 b.

10000007000000000003

Q3- a

(i)

$$\begin{bmatrix} 1/3 & 2/3 & 0 \\ 0 & 2/3 & 1/3 \end{bmatrix}$$

(ii)

$$H(X) = -\sum_k p_k \, log_2(p_k)$$

$$H(X) = -\left[\frac{1}{4} \, log_2\left(\frac{1}{4}\right) + \frac{3}{4} \, log_2\left(\frac{3}{4}\right)\right] = 0.811$$

(iii)

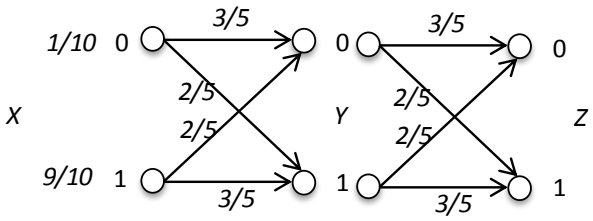
p(Y=1|X=0) = 0

p(Y=0,X=1) = 0

p(Y=U) = (1/4)(2/3)+(3/4)(2/3) = 2/3

Q3- b

(i)



(ii)

p(Y=1|X=0) = $\frac{2}{5}$

p(Y=1,X=0) = (1/10)(2/5) = 1/25

p(Y=1) = (1/10)(2/5)+(9/10)(3/5) = 29/50

(iii)

p(Z=0|X=0) = (3/5)(3/5) + (2/5)(2/5) = 13/25

p(Z=1,X=1) = (9/10)(2/5)(2/5) + (9/10)(3/5)(3/5) = 117/250

p(Z=1) = (9/10)(3/5)(3/5) + (9/10)(2/5)(2/5) + (1/10)(2/5)(3/5) + (1/10)(3/5)(2/5)= 129/250

Q4- a

$$d_{min} = 5$$

$$d_d = d_{min} - 1 = 4$$

$$d_c = \frac{d_{min} - 1}{2} = 2$$

Q4- b

(i)

$H = [-P^T | I_{n-k}] =$

$$\begin{matrix} 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \end{matrix}$$

(ii)

$$c\,H^T = \quad [1\,0\,1\,1\,0\,1\,0\,0\,0\,1\,0\,1\,0\,0\,1] \quad \begin{matrix} \mathbf{1} & \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{1} \\ \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{1} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{1} \\ \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{0} \\ \mathbf{1} & \mathbf{1} & \mathbf{0} & \mathbf{1} \\ \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{1} \\ \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} \end{matrix} \quad = 1\,1\,1\,0$$

Error vector = 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0

(iii) Codeword = 1 0 1 1 0 1 1 0 0 1 0 1 0 0 1